$\mu \to e\gamma$ AT A RATE OF ONE OUT OF 10^9 MUON DECAYS?

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It is proposed that lepton number conservation, purely left-handed charged weak currents and vanishing neutrino masses are a limiting case of a parity symmetric $SU2_L \times SU_R \times U2^V$ gauge theory. Right-handed neutrinos acquire a lepton number violating mass, leaving an $SU2_L \times U1$ subgroup unbroken. Consequences for the decay $\mu \to e\gamma$ are studied.

In a gauge theory of weak and associated interactions (electromagnetic, strong, ...) in which parity and time reversal are spontaneously broken discrete symmetries, on an equal footing with the continuous symmetries of SUn (fermions) $\times$ SUn (fermions) corresponding to chiral substitutions of lepton and quark flavors respectively, the following features particular to leptons are intrinsically related:

(i) Neutrino masses are nonvanishing but orders of magnitude are smaller than charged lepton masses ‡1 [1].

(ii) Considering the mass scale set by $G_F^{-1/2} \approx 300$ GeV as small compared to the mass scale on which parity invariance is restored, the chiral structure of the (light) neutrinos determines the corresponding (lowest level) gauge group to be $((SU2)_L \times U1)$ [3].

(iii) Right-handed leptons (and quarks) are singlets with respect to $(SU2_L)$ [3]. (Scalars generating the masses of the charged leptons transform as doublets under $SU2_L$.)

(iv) Lepton numbers $(L_e, L_\mu, L_e + L_\mu)$ are not exactly conserved [4–6]. We study a scheme for leptons in which the breakdown of parity is reflected by the sequence of gauge groups

$$\underbrace{SU2_L \times SU2_R}_{SO4} \times (U1)^V \to SU2_L \times (U1). \qquad (1)\ [7]$$

We will investigate the consequences for the decays

$$\mu \to e^\gamma\ (\mu \to 3e). \qquad [8]\ ‡2$$

An irreducible (lepton) multiplet (one electronlike, one muonlike, . .) can be represented alternatively by 4 Majorana fields or 4 (e.g.) left-handed fields (including *CPT*-conjugate fields):

$$\psi_A^{(\nu eL)} \leftrightarrow (\nu_e)_L^{\dot\alpha}, \quad \psi_A^{(e\bar L)} \leftrightarrow (e^-)_L^{\dot\alpha}, \quad \psi_A^{(N_{eR})} \leftrightarrow (\widetilde{N}_e)_L^{\dot\alpha} (= \epsilon^{\dot\alpha\dot\beta} N^*_{e_{R\dot\beta}}), \quad \psi_A^{(e\bar R)} \leftrightarrow (\widetilde{e}^+)_L^{\dot\alpha} (= \epsilon^{\dot\alpha\dot\beta} e^{-*}_{R_{\dot\beta}}). \qquad (2)$$

In eq. (2) $\psi_A^{(k)}$ $(k_1 A = 1, ... , 4)$ denote hermitian (four component) Majorana fields, $(\ell)_L$ denote left-handed, com-

‡1 In the light of the present discussion we note that the vectorlike scheme of ref [1] is unable to correlate points (i)–(iv). It is also at variance with the inclusive neutrino-hadron (isoscalar target) cross section ratio
$\sigma^{\bar\nu N \to \bar\nu X}/\sigma^{\nu N \to \nu X} = \begin{cases} 0.59 \pm 0.14 & \text{GGM [2]} \\ \leqslant 0.61 \pm 0.25 & \text{HPNF [2]} \end{cases}$, and with the elastic neutrino-proton cross section ratio $\sigma^{\bar\nu p \to \bar\nu p}/\sigma^{\nu p \to \nu p}$ $= 0.4 \pm 0.2$ HPWF [2].

‡2 The experiment currently in progress at SIN looking for the decay $\mu^+ \to e^+\gamma$ exploiting the high intensity muon beam has triggered a renewed discussion of possible sources of lepton number violations [7]. We should like to stress that we take the rumors of a positive signal as stimulus for the present investigation only.

plex two component fields ($\dot{\alpha} = 1, 2$; $\ell = \nu_e, \tilde{N}_e, e^-, \tilde{e}^+$ and $\epsilon^{\dot{\alpha}\dot{\beta}} \equiv \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ [‡3]. The (U1) weak hypercharge Y_V takes on the values -1 for $(\nu_e, e^-)_L$; $(N_e, e^-)_R$ in order to satisfy the relation

$$Q^{\text{e.m.}} = I_3^L + I_3^R + \tfrac{1}{2} Y_V \,, \tag{3}$$

and thus can be identified with (-1) lepton number [‡4].

Yukawa couplings can only involve scalar multiplet transforming like the symmetric product of two spinor representations of SO4, i.e. the multiplets

$$\varphi_L: \quad (3, 1, \pm 2), \qquad \varphi_R: \quad (1, 3; \mp 2), \qquad \varphi_{LR}: \quad (2, 2; 0)\,. \tag{4}$$

In eq. (4) the first two numbers denote the values of $(2J_{L,(R)} + 1)$ characterizing the $SU2_{L,R}$ multiplets and the third number gives the value of Y_V.

The multiplicities of the representations in eq. (4) occurring in the scalar sector are of physical significance. They determine the range of Yukawa interactions beyond the couplings of the Goldstone mode scalars to fermions. These "genuine" Yukawa interactions will in general violate at least some of the conservation laws valid for the (incomplete) interactions of fermions and gauge bosons [10].

The hierarchy of gauge groups (1) extended to include the quarks (left-handed doublets, right-handed singlets)

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \begin{pmatrix} c \\ s' \end{pmatrix}_L ; \qquad (u, d, c, s)_R ; \qquad d'_L = \cos\vartheta_c d + \sin\vartheta_c s, \qquad s'_L \perp d'_L \,, \tag{5}$$

would imply the following unacceptable mass matrix provided only one multiplet $\varphi_{LR}(2, 2; 0)$ would generate all quark masses:

$$m_u/m_d = m_c/m_s ; \qquad \vartheta_c = 0\,. \tag{6}$$

We conclude that there are several scalar multiplets transforming like $\varphi_{LR}(2, 2, 0)$ and consider for simplitiy two such multiplets coupling to the leptons together with two multiplets transforming like $\varphi_R(1, 3, -2)$ and $\varphi_L(3, 1, -2)$ (and their antiparticles):

$$\begin{gathered} \varphi_{LR}: \quad a_{\rho\sigma} = \begin{pmatrix} a_1 & a^+ \\ a^- & a_2 \end{pmatrix}; \qquad a^*_{\rho\sigma}; \qquad b_{\rho\sigma} = \begin{pmatrix} b_1 & b^+ \\ b^- & b_2 \end{pmatrix}, \qquad b^*_{\rho\sigma} \\ \varphi_R: \quad d_{\rho\sigma} = d_{\sigma\rho} = \begin{pmatrix} d_0 & -\frac{1}{\sqrt{2}} d^+ \\ -\frac{1}{\sqrt{2}} d^+ & d^{++} \end{pmatrix}; \quad d^*_{\rho\sigma}, \qquad \varphi_L: \quad S_{\rho\sigma} = S_{\sigma\rho} = \begin{pmatrix} S_0 & -\frac{1}{\sqrt{2}} s^+ \\ -\frac{1}{\sqrt{2}} s^+ & s^{++} \end{pmatrix}; \quad S^*_{\rho\sigma}\,. \end{gathered} \tag{7}$$

$a_{1,2}$, $b_{1,2}$, d_0, S_0 are electrically neutral but not self-conjugate; d^+, s^+ singly and d^{++}, s^{++} doubly charged.

The interaction of the scalars in (7) with the electron and muon multiplets [‡5] are:

‡3 The relation between Majorana and chiral fields is. $\psi^{(k)}_{1,2} = R^{(k)}_{1,2}$; $\psi^{(k)}_{3,4} = J^{(k)}_{1,2}$, $R^{(k)}_{1,2} = \ell^{\dot{1},\dot{2}}_{kL} + \text{h.c.}$, $J^{(k)}_{1,2} = (1/i)\ell^{\dot{1},\dot{2}}_{kL} + \text{h.c.}$

‡4 Extending Y_V to quarks. ($Y_V(q) = \frac{1}{3}$, $Y_V(q) = -\frac{1}{3}$) implies $Y_V = A - L$, where A denotes baryon number and $L = L_e + L_\mu +$ (overall) lepton number. Thus Y_V is conserved if baryon and lepton numbers are separately conserved, and yet there does not exist a long range force associated with the corresponding boson. This problem generally arises in unified gauge theories [9].

‡5 We do not include for simplicity an eventual third lepton multiplet including the heavy charged lepton which may have been observed in the $e^+e^- \to e^-\mu^+X$ events at Spear [11] involving no (detected) hadron in the field state.

$$\mathcal{H}_{\mathrm{Y}} = \begin{Bmatrix} \bar{\ell}^{(i)}_{\rho} \, [h_1^{ik} \mathrm{a}_{\rho\sigma} + h_2^{ik} \mathrm{b}_{\rho\sigma}] \, \dfrac{1+\gamma_5}{2} \, \ell^{(k)}_{\sigma} \\ \ell^{(i)\alpha}_{\rho \mathrm{R}} \, \tfrac{1}{2} h_{\mathrm{R}}^{ik} \mathrm{d}_{\rho\sigma} \ell^{(k)}_{\sigma \mathrm{R}\alpha} \\ \ell^{(i)}_{\rho \mathrm{L}\dot{\beta}} \, \tfrac{1}{2} h_{\mathrm{L}}^{ik} \mathrm{s}_{\rho\sigma} \ell^{(k)\dot{\beta}}_{\sigma \mathrm{L}} \end{Bmatrix} + \mathrm{h.c.}$$

$$\ell_1^{(1)} = (\nu_{\mathrm{e}}, \mathrm{N_e}); \qquad \ell_2^{(1)} = \mathrm{e}^-; \qquad \ell_1^{(2)} = (\nu_\mu, \mathrm{N}_\mu); \qquad \ell_2^{(2)} = \mu^- . \tag{8}$$

Spontaneous symmetry breaking is induced by the vacuum expectation values

$$\langle \mathrm{a} \rangle_0 = \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix}, \qquad \langle \mathrm{b} \rangle_0 = \begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix}, \qquad \langle \mathrm{d} \rangle_0 = \begin{pmatrix} D & 0 \\ 0 & 0 \end{pmatrix}, \qquad \langle \mathrm{s} \rangle_0 = \begin{pmatrix} S & 0 \\ 0 & 0 \end{pmatrix} . \tag{9}$$

We may assume A_1, A_2, B_1, D real, nonnegative without loss of generality. The breakdown $\mathrm{SU2_L \times SU2_R \times U1_V} \to \mathrm{SU2_L \times (U1)}$ correspond to $D \neq 0$ only.

Points (i)–(iv) become indeed logically connected if we choose

$$S \approx 0; \qquad (A_{1,2}; B_{1,2}) = \mathrm{O}(G_{\mathrm{F}}^{-1/2}); \qquad (A_{1,2}; B_{12}) \ll D. \tag{10}$$

The mass matrix in the charged lepton sector $\mathcal{M}_{\mathrm{ch}}$ is independent of D and it serves to define the basic fields with respect to which $\mathcal{M}_{\mathrm{ch}}$ is diagonal

$$\begin{pmatrix} \nu_{\mathrm{e}} \\ \mathrm{e}^- \end{pmatrix}_{\mathrm{L}}, \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_{\mathrm{L}}; \quad \begin{pmatrix} \mathrm{N_e} \\ \mathrm{e}^- \end{pmatrix}_{\mathrm{R}}, \quad \begin{pmatrix} \mathrm{N}_\mu \\ \mu^- \end{pmatrix}_{\mathrm{R}} . \tag{11}$$

In the above basis the mass matrices in the neutral and charged lepton sectors are, before D is turned on

$$\mathcal{M}_{\mathrm{n}} = A_1 h_1 + B_1 h_2; \qquad \mathcal{M}_{\mathrm{ch}} = A_2 h_1 + B_2 h_2 = \begin{pmatrix} m_{\mathrm{e}} & 0 \\ 0 & m_\mu \end{pmatrix}. \tag{12}$$

m_{e}, m_μ in eq. (12) are the physical masses of electron and muon respectively.

The 7 gauge bosons corresponding to $\mathrm{SU2_L \times SU2_R \times (U1)_v}$ shall be denoted by

$$(\mathrm{W}^+_\mu)_{\mathrm{L,R}} = \frac{1}{\sqrt{2}} (\mathrm{W}^1_\mu - \mathrm{i}\mathrm{W}^2_\mu)_{\mathrm{L,R}}; \qquad (\mathrm{W}^0_\mu)_{\mathrm{L,R}} = (\mathrm{W}^3_\mu)_{\mathrm{L,R}}; \mathrm{Y}^{\mathrm{V}}_\mu . \tag{13}$$

Using the basis of eq. (11) the lepton currents couple to $(\mathbf{W}_{\mathrm{L,R}}, Y_{\mathrm{V}})$ conserving parity invariance:

$$\mathcal{G} = \sum_{k=1}^{2} \begin{Bmatrix} \bar{\ell}^{(k)} \gamma^\mu g \dfrac{\tau}{2} \left[\dfrac{1+\gamma_5}{2} \mathbf{W}^{\mathrm{L}}_\mu + \dfrac{1-\gamma_5}{2} \mathbf{W}^{\mathrm{R}}_\mu \right] \ell^{(k)} \\ \\ - \dfrac{g_{\mathrm{V}}}{2} \bar{\ell}^{(k)} \gamma^\mu \mathrm{Y}^{\mathrm{V}}_\mu \ell^{(k)} \end{Bmatrix} . \tag{14}$$

In the limit $D \to \infty$ the above scheme reduces to the $\mathrm{SU2_L \times U1}$ ($\mathrm{U1} \neq \mathrm{(U1)_V}$) theory with its symmetry breaking provided by a set of scalar $\mathrm{SU2_L}$-doublets as proposed by Weinberg [3] with the identification

$$\sin^2\vartheta_W = \frac{g_V^2}{g^2+2g_V^2}\left(\leqslant\tfrac{1}{2}\right); \qquad \sqrt{4\pi\alpha} = e = g\sin\vartheta_W\,, \tag{15}$$

$$\begin{array}{ll} Y = \sin\vartheta_V W_R^0 + \cos\vartheta_V Y^V & \\ \mathrm{tg}\,\vartheta_V = \dfrac{g_V}{g}\,; \qquad \mathrm{tg}\,\vartheta_W = \sin\vartheta_V & \rightarrow \\ m_{Z'}^2 = 2(g^2+g_V^2)D^2 \to \infty & \end{array} \left\{ \begin{array}{l} \gamma = \sin\vartheta_W W_L^0 + \cos\vartheta_W Y \\ Z = \cos\vartheta_W W_L^0 - \sin\vartheta_W Y \\ (Z' = \cos\vartheta_V W_R^0 - \sin\vartheta_V Y^V)\,. \end{array}\right.$$

The mass matrices of the gauge bosons in the neutral and charged sectors are given in terms of the quantities

$$F^2 = \tfrac{1}{2}(|A_1|^2+|A_2|^2+|B_1|^2+|B_2|^2)\,, \qquad G = \mathrm{Re}(A_1^*A_2 + B_1^*B_2) = F^2\sin\varphi; \quad D^2\,, \tag{16}$$

in the following form

$$\mathcal{L}_{m_B^2}^{\text{neutral}} = \tfrac{1}{2}g^2F^2(W_L^0 - W_R^0)^2 + D^2(gW_R^0 - g_V Y^V)^2\,,$$
$$\mathcal{L}_{m_B^2}^{\text{charged}} = g^2\{F^2W_L^-W_R^+ - G(W_L^-W_R^+ + W_R^-W_L^+) + (F^2+D^2)W_R^-W_R^+\}\,. \tag{17}$$

Expanding in powers of F^2/D^2 we obtain the eigenstates of the mass matrix ν:

$$Z_1 \approx Z + \xi Z'\,, \qquad Z_2 \approx \xi Z + Z'\,,$$
$$m_{Z_1}^2 = \frac{g^2F^2}{\cos^2\vartheta_W}\left(1+O\left(\frac{F^2}{D^2}\right)\right), \qquad m_{Z_2}^2 = \frac{2g^2D^2}{1-\mathrm{tg}^2\vartheta_W}\left(1+O\left(\frac{F^2}{D^2}\right)\right), \qquad \xi = \frac{m_{Z_1}^2}{m_{Z_2}^2}\left(\frac{1-\mathrm{tg}^2\varphi_W}{1+\mathrm{tg}^2\vartheta_W}\right)^{1/2}, \tag{18}$$

in the neutral sector and

$$W_1^+ \approx W_L^+ + \eta W_R^+; \qquad W_2^+ \approx W_R^+ - \eta W_L^+, \qquad \eta = m_{W_1}^2 G/m_{W_2}^2F^2 = m_{W_1}^2\sin\varphi/m_{W_2}^2$$

$$m_{W_1^\pm}^2 = g^2F^2\;(1+O(F^2/D^2)), \quad m_{W_3^\pm}^2 = g^2D^2\;(1+O(F^2/D^2)), \tag{19}$$

and thus $F = (4\sqrt{2}G_F)^{-1/2} \approx 124$ GeV.

We note the relation

$$m_{W_1^\pm}^2/\cos^2\vartheta_W m_{Z_1}^2 = 1 + O(F^2/D^2)\,, \tag{20}$$

which reflects the realization of the doublet breaking mechanism inherent to $SU2_L \times U1$ in the limit $F/D \to 0$.

We now turn to the neutrino mass matrix which contains the lepton number conserving part

$$(\mathcal{M}_n)_{ik} = A_1h_1^{ik} + B_1h_2^{ik}\,,$$

and a lepton number violating part

$$(N_R)_{ik} = Dh_R^{ik} = (N_R)_{ki}\,.$$

In the basis we have chosen h_1, h_2 are constrained by eq. (12) but this does not constrain $(\mathcal{M}_n)$.

Combining the *CPT*-transformed right-handed – with the left-handed fields in eq. (11) we obtain the Majorana-like basis of eq. (2)

$$(f^{\dot\alpha}) = (f_1^{\dot\alpha}, f_2^{\dot\alpha}, \tilde{f}_1^{\dot\alpha}, \tilde{f}_2^{\dot\alpha}) = (\nu_{eL}, \nu_{\mu L}, \tilde{N}_{eL}, \tilde{N}_{\mu L})^{\dot\alpha}, \quad \dot\alpha = 1, 2.$$

The mass eigenstates correspond to the solutions of the equation

$$(i\partial_0 + \frac{1}{i}\nabla\sigma) f = \mu(i\sigma_2) f^* , \tag{21}$$

where μ is the 4×4 symmetric (complex) matrix of the following 2×2 block form

$$\mu = \begin{pmatrix} 0 & \mathcal{M}_n \\ \mathcal{M}_n^T & N_R \end{pmatrix} . \tag{22}$$

We assume the strength's of the Yukawa couplings h_1, h_2, are of the same order of magnitude or smaller than h_R

$$O(|h_1|) = O(|h_2|) \leqslant O(|h_R|), \qquad |h|^2 = \sum_{i,k} |h_{ik}|^2 .$$

It follows

$$O(|\mathcal{M}_n|) = O(|\mathcal{M}_{ch}|) \leqslant O\ \frac{F}{D} |N| \ll |N| , \tag{23}$$

and hence we can diagonalize μ expanding with respect to $|\mathcal{M}_n|/|N_R|$:

$$\mu = v\mu_{diag} v^T ; \qquad vv^+ = \mathbf{1}, \qquad f = vf_{(1)}, \qquad \mu_{diag} = \begin{pmatrix} m_1^\nu & & & \\ & m_2^\nu & & 0 \\ & 0 & m_1^N & \\ & & & m_2^N \end{pmatrix} \quad m_{1,2}^\nu, m_{1,2}^N \geqslant 0 . \tag{24}$$ [‡6]

From eq. (24) we infer:

$$m_\nu = O\left(\frac{|\mathcal{M}_n|^2}{m_{1,2}^N}\right) \lesssim 10 \text{ eV} . \tag{25}$$

Assuming $m^\nu/m_\mu = O(10^{-8})$ it follows

$$\left(10^4 \frac{|\mathcal{M}_n|}{m_\mu}\right)^2 = O\left(\frac{m_N}{m_\mu}\right), \qquad |\mathcal{M}|_n \approx m_\mu \to m_N \approx 10^7 \text{ GeV}, \qquad |\mathcal{M}|_n \approx 10^{-3} m_\mu \to m_N \approx 10 \text{ GeV}. \tag{26}$$

We proceed to compute and estimate the decay amplitudes for the process $\mu^- \to e^-\gamma$ due to fermion-gauge boson interactions alone.

The corresponding radiation graph [12] yields the following amplitude

‡6 Reducing all four by four matrices to 2×2 block forms we obtain: $v = \begin{pmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{pmatrix}$: $v_{11}\begin{pmatrix} m_1^\nu & 0 \\ 0 & m_2^\nu \end{pmatrix} v_{11}^T \approx -\mathcal{M}_n N_R^{-1} \mathcal{M}_n^T$,

$v_{22}\begin{pmatrix} m_1^N & 0 \\ 0 & m_2^N \end{pmatrix} v_{22}^T \approx N_R$, $\quad v_{12} \approx \mathcal{M}_n N_R^{-1} v_{22}$; $\quad v_{21} \approx -(\mathcal{M}_n N_R^{-1})^+ v_{11}$.

Table 1
Order of magnitude of helicity amplitudes in the four cases, I: $m_N \ll m_{W_1}$, II: $m_N \approx m_{W_1}$, III: $m_N \approx m_{W_2}$, IV: $m_N \gg m_{W_2}$.

	$\lvert T_{LL}\rvert$	$\lvert T_{LR}\rvert = O(\lvert T_{RL}\rvert)$	$\lvert T_{RR}\rvert$
I:	$\frac{1}{4}\times 10^{-8}\,\frac{m_\mu m_N}{m_{W_1}^2}$	$\sin\varphi \times 10^{-4}\left(\frac{m_N}{m_\mu}\right)^{1/2}\,\frac{m_{W_1}^2}{m_{W_2}^2}$	$\frac{1}{4}\,\frac{m_{W_1}^2}{m_{W_2}^2}\,\frac{m_N^2}{m_{W_2}^2}$
II:	$10^{-8}\,\frac{m_\mu}{m_{W_1}}$	$\sin\varphi \times 10^{-4}\left(\frac{m_{W_1}}{m_\mu}\right)^{1/2}\,\frac{m_{W_1}^2}{m_{W_2}^2}$	$\frac{1}{4}\left(\frac{m_{W_1}}{m_{W_2}}\right)^4$
III:	$\frac{1}{3}\times 10^{-8}\,\frac{m_\mu}{m_{W_2}}$	$\sin\varphi \times 10^{-4}\left(\frac{m_{W_2}}{m_\mu}\right)^{1/2}\,\frac{m_{W_1}^2}{m_{W_2}^2}$	$\frac{m_{W_1}^2}{m_{W_2}^2}$
IV	$\frac{1}{3}\times 10^{-8}\,\frac{m_\mu}{m_N}$	$\sin\varphi \times 10^{-4}\left(\frac{m_N}{m_\mu}\right)^{1/2}\,\frac{m_{W_1}^2}{m_{W_2}^2}$	$\frac{m_{W_1}^2}{m_N^2}\log\left(\frac{m_N^2}{m_{W_2}^2}\right)$

$$T_{\mu\to e\gamma} = \frac{1}{m_{W_1}^2}\,\bar{u}_e i\sigma_{\alpha\beta}\epsilon_\gamma^\alpha k_\gamma^\beta T_W u_\mu, \qquad T_W = \frac{eg^2}{32\pi^2}\left(A\,\frac{1-\gamma_5}{2} + B\,\frac{1+\gamma_5}{2}\right). \tag{27}$$

The factor $1/m_{W_1}^2$ in eq. (27) simplifies the comparison with the (main) $\mu^- \to e^-\bar{\nu}_e\nu_\mu$ decay.
In the limit $m_e \to 0$ we obtain

$$A = \sum_k (T_{LL}^k + T_{LR}^k); \qquad B = \sum_k (T_{RR}^k + T_{RL}^k)\,^{‡7}, \qquad k = (\nu_1, \nu_2, N_1, N_2) \quad (m_{\nu_1}, m_{\nu_2}, m_{N_1}, m_{N_2}),$$
$$m_N = \operatorname*{Max}_{1,2}(m_{N_{1,2}}) \tag{28}$$

k (= 1, ... , 4) in eq. (28) denotes the contribution of a given mass eigenstate in the neutral fermion sector.
We consider four cases, I: $m_N \ll m_{W_1}$ ($\ll m_{W_2}$); II: $m_N \approx m_{W_1}$; III: $m_{W_1} \ll m_N \approx m_{W_2}$; IV: $m_N \gg m_{W_2}$.
Within the above limits exact results can be obtained in terms of the mass matrix μ in eq. (24) ‡7. We only give the respective orders of magnitudes of the amplitudes $T_{xy} = \Sigma_k T_{xy}^k$ here in terms of the parameters m_{N_1}, $m_{W_1} \approx 37.6$ GeV/$\sin\vartheta_W \approx 60$ GeV

$$|\mathcal{M}_n| \approx (m_\nu m_N)^{1/2} \quad (m_\nu\text{: characteristic neutrino mass}).$$

We assume $m_\nu \approx 10^{-8}\,m_\mu$ here ($|\mathcal{M}_n| \approx 10^{-4}\,(m_\mu m_N)^{1/2}$). The results are given in table 1.
The branching ratio $B^{\mu\to e\gamma} = \Gamma^{\mu\to e\gamma}/\Gamma^{tot}$, ($\Gamma_\mu^{tot} \approx G_F^2 m_\mu^5/192\pi^3$) restricts the possibilities in table 1:

‡7 T_{xy}^k in eq. (28) are determined from the mass matrix μ in eq. (24). $T_{xy}^k = \sum_{1,2} c_x^k(e)\bar{c}_y^k(\mu)t_{xy}^k(m_\mu, m_k; m_{W_{1,2}})$, $c_L^k(e^-) = v_{1k}$, $c_L^k(\mu^-) = v_{2k}$, $c_R^k(e^-) = \bar{v}_{3k}$, $c_R^k(\mu^-) = \bar{v}_{4k}$, $t_{LL}^k \approx t_{LL}(m_k, m_{W_1})$. $t_{LR}^k \approx t_{RL}^k \approx -\eta\,\frac{m_k}{m_\mu}\,t_{LR}(m_k, m_{W_1})$, $t_{RR}^k \approx t_{LL}(m_k, m_{W_2})$; $\eta = \sin\varphi/m_{W_2}^2$, $t_{LL}(a,b) = 2I_1 + \frac{a^2}{b^2}(J_3 - J_1)$; $t_{LR}(a,b) = 4I_{RL} + \frac{a^2}{b^2}J_3$, $\{I_1, I_{RL}, J, J_3\} = I_\kappa$, $\kappa = 1, \ldots, 4$, $I_\kappa(a,b) = \int_0^1 dx\, f_\kappa(x)\,\frac{m_{W_1}^2}{F}$; $F(a,b;x) = (1-x)b^2 + xa^2$, $f(I_1) = (1-x)^2(1-\frac{1}{2}x)$, $f(I_{RL}) = (1-x)^2$, $f(J_1) = \frac{1}{2}x(1-x)^2$, $f(J_3) = x(1-x)$.

$$\frac{2\pi}{3\alpha} B^{\mu\to e\gamma} = (|A|^2 + |B|^2)\,; \qquad A = T_{\rm LL} + T_{\rm LR}, \qquad B = T_{\rm RR} + T_{\rm RL}\,. \tag{29}$$

If $B^{\mu\to e\gamma} = O(10^{-9})$ then Max $(|A|, |B|) = O(5 \times 10^{-4})$. This excludes $T_{\rm LL}$ grom giving a sizable contribution.

For $T_{\rm LR}$, $T_{\rm RL}$ I–III would imply $\sin\varphi(m_{\rm W_1}/m_{\rm W_2})^2 \approx (0.1\text{–}0.2)$ and thus considerable deviation of the lepton current from the V–A structure. We conclude that $T_{\rm LR}$, $T_{\rm RL}$ cannot contribute significantly (always for $B^{\mu\to e\gamma} \approx 10^{-9}$) unless $m_{\rm N} \gg m_{\rm W_2}$ (e.g. $\sin\varphi\, m_{\rm W_1}^2/m_{\rm W_2}^2 = 10^{-2} \to m_{\rm N} \approx 2 \times 10^4$ GeV).

Finally $T_{\rm RR}$ can be a source of $\mu \to e\gamma$ decay at the above level in cases III and IV provided $m_{\rm N} \leqslant 10\, m_{\rm W_2}$.

In conclusion we remark that III is an outstanding possibility (unless $B^{\mu\to e\gamma} \ll 10^{-9}$), yielding the parameters

$$m_{\rm N} \approx m_{\rm W_2} \approx 45\, m_{\rm W_1} \approx 2700 \text{ GeV}\ ^{\ddagger 8}, \qquad |T_{\rm LL}|, |T_{\rm LR}|, |T_{\rm RL}| \ll |T_{\rm RR}|\,.$$

The $\mu \to 3e$ decay corresponds in all cases considered here, mainly to ordinary Dalitz pairs.

A dominating $T_{\rm RR}$ amplitude would produce predominantly right-handed e^- from $\mu^- \to e^-\gamma$ decay ($\mathbf{P}_{\rm long}(e^-) = +v_e/c$) and no T-violating polarization of e^- perpendicular to the plane spanned by the spin of μ^- and the momentum of e^-. This polarization could be large provided $|T_{\rm RL}|, |T_{\rm LR}| = O(|T_{\rm RR}|)$. ‡9

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‡8 The Majorana leptons N couple to e^- and thus produce neutrinoless double β-decay. However, their coupling constant is reduced relative to the Fermi constant by a factor $(m_{\rm W_1}/m_{\rm W_2})^2 \approx 1/2 \times 10^{-3}$. This together with $m_{\rm N} \approx m_{\rm W_2}$ insures an effect small enough to be compatible with observation [13].

‡9 The asymmetry parameters of the $\mu^- \to e^-\gamma$ decay are $\vartheta = \vartheta(\boldsymbol{P}_{e^-}, \boldsymbol{S}_\mu)$, $\quad \dfrac{1}{\Gamma_{\mu\to e^-\gamma}}\dfrac{d\Gamma}{d\cos\vartheta} = \tfrac{1}{2}(1 + \mathbf{P}_\mu - a\cos\vartheta)$,

$$a = \frac{|A|^2 - |B|^2}{|A|^2 + |B|^2}, \qquad \frac{d\left[\Gamma \,\mathrm{Det}\left(\boldsymbol{S}_{e^-}, \boldsymbol{S}_{\mu^-}, \dfrac{\boldsymbol{P}_e}{|\boldsymbol{P}_e|}\right)\right]/d\cos\vartheta}{d\Gamma/d\cos\vartheta} = \mathbf{P}_{\mu^-}\frac{\sin\vartheta}{1+\mathbf{P}_{\mu^-} a\cos\vartheta}\,\frac{2\,\mathrm{Im}\,\bar{A}B}{|A|^2+|B|^2}, \qquad CPT\ \ e^- \to e^+, \mu^- \to \mu^+,$$

$1 \pm \gamma_5 \to 1 \mp \gamma_5$, $\quad A \to +\bar{B}, B \to \bar{A}$, $\quad \mathbf{P}_{\mu^-}$ degree of polarization of the stopped muon.

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